



**LEBANESE AMERICAN UNIVERSITY**  
**Department of Computer Science and Mathematics**

**MTH101 – Calculus I**  
**Spring 2014**  
**Exam 1**  
**(March 17, 2014)**

**NAME:** Answer Key **ID:** \_\_\_\_\_

**Duration:** 60 minutes

**Instructor:** Ms. Liwa Sleiman

**This exam is comprised of 8 problems. Answer the questions in the space provided for each problem. If more space is needed, use the back of the page. Make sure to justify all your answers.**

Problem	Grade points
I	12 %
II	12 %
III	09 %
IV	10 %
V	18 %
VI	08 %
VII	16 %
VIII	15 %
<b>Total</b>	<b>100</b>

I. Consider the points A(-3,2) and B(1,-2).

a) (6%) Find the equation of the line passing through A and B. (1)

$$\text{slope} = m = \frac{y_B - y_A}{x_B - x_A} = \frac{-2 - 2}{1 + 3} = \frac{-4}{4} = -1$$

$$(2) y - y_A = m(x - x_A)$$

$$(1) y - 2 = -1(x + 3)$$

$$y - 2 = -x - 3 \Rightarrow \boxed{y = -x - 1}$$

b) (6%) Write the equation of the circle with center A and radius 5.

$$(x - x_A)^2 + (y - y_A)^2 = R^2$$

$$(x + 3)^2 + (y - 2)^2 = 25 \quad (2)$$

II. Find the domain of the following functions:

a) (5%)  $f(x) = \frac{5x-2}{(x-3)(x+1)}$

$$(x-3)(x+1) \neq 0 \quad (1)$$

$$x-3 \neq 0 \quad \text{or} \quad x+1 \neq 0 \quad (1)$$

$$(1) x \neq 3$$



$$D = \mathbb{R} - \{3, -1\} = \boxed{\mathbb{R}} = (-\infty, -1) \cup (-1, 3) \cup (3, +\infty) \quad (2)$$

b) (7%)  $g(x) = \frac{\sqrt{3-x}}{x+5}$

2 conditions

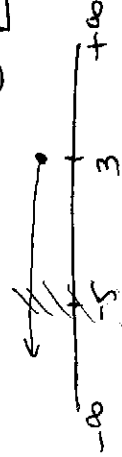
$$(1) 3-x \geq 0$$

$$-x \geq -3$$

$$(1) \boxed{x \leq 3}$$

$$x+5 \neq 0 \quad (1)$$

$$\boxed{x \neq -5} \quad (1)$$



$$D = (-\infty, -5) \cup (-5, 3] \quad (3)$$

III. Determine whether the following functions are even, odd, or neither. Justify your answer.

a) (5%)  $f(x) = x^3 - x$

$$f(-x) = (-x)^3 - (-x) = -x^3 + x = -(x^3 - x) = -f(x) \quad \text{odd} \quad \textcircled{1}$$

b) (4%)  $g(x) = |x - 2|$

$$g(-x) = |-x - 2| \neq f(x) \quad \text{not even} \quad \textcircled{2}$$

$$\neq -f(x) \quad \text{not odd} \quad \textcircled{2}$$

IV. Consider the functions

$$f(x) = \frac{5}{2-x}, \quad g(x) = x^2 - 1, \quad \text{and} \quad h(x) = \sqrt{x+3}$$

a) (3%) Find  $g \circ f(3)$ .

$$g \circ f(3) = g[f(3)] = g(-5) = 25 - 1 = 24 \quad \textcircled{1}$$

b) (7%) Write a formula for  $f \circ g \circ h(x)$ .

$$\begin{aligned} f \circ g \circ h(x) &= f[g(h(x))] \quad \textcircled{1} \\ &= f[g(\sqrt{x+3})] \quad \textcircled{1} \\ &= f[(\sqrt{x+3})^2 - 1] \quad \textcircled{1} \\ &= f(x+3-1) \quad \textcircled{1} \\ &= f(x+2) \quad \textcircled{1} \\ &= \frac{5}{2-(x+2)} = \frac{-5}{x} \quad \textcircled{1} \end{aligned}$$

V. a) (8%) Find  $\sin^2 \frac{\pi}{12}$  (without using your calculator)

③  $\cos 2x = 1 - 2 \sin^2 x$

$x = \frac{\pi}{12}$

$\sin^2 \frac{\pi}{12} = \frac{1 - \cos \frac{\pi}{6}}{2}$  ①

$= \frac{1}{2} - \frac{\sqrt{3}}{4}$  ①

②  $\cos 2\frac{\pi}{12} = 1 - 2 \sin^2 \frac{\pi}{12}$

①  $\cos \frac{\pi}{6} = 1 - 2 \sin^2 \frac{\pi}{12}$

b) (10%) Given  $\tan x = 4$ ,  $x \in [\pi, \frac{3\pi}{2}]$ .

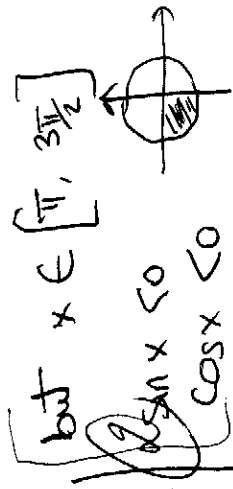
Find  $\sin x$  and  $\cos x$ .

$-\tan x = 4$

①  $\frac{1}{\cos^2 x} = \sec^2 x = 1 + \tan^2 x = 1 + 16 = 17$  ①

$\cos^2 x = \frac{1}{17}$  ①

$\cos x = \pm \sqrt{\frac{1}{17}}$  ①



$\cos x = -\sqrt{\frac{1}{17}}$  ①

$\sin x = \cos x \cdot \tan x$  ①

$= -\sqrt{\frac{1}{17}} \cdot 4$

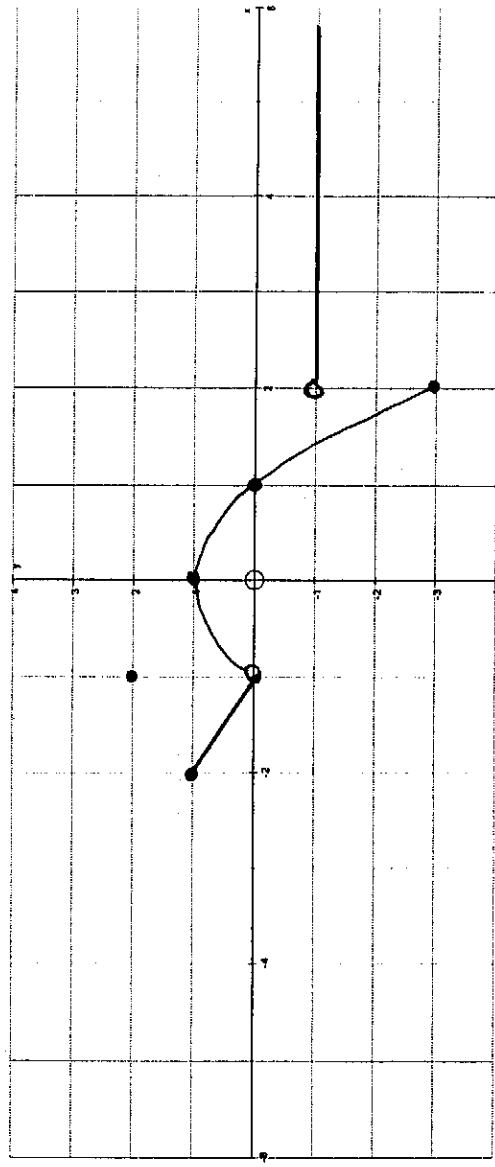
$= -\frac{4}{\sqrt{17}}$  ①

VI. (8%)

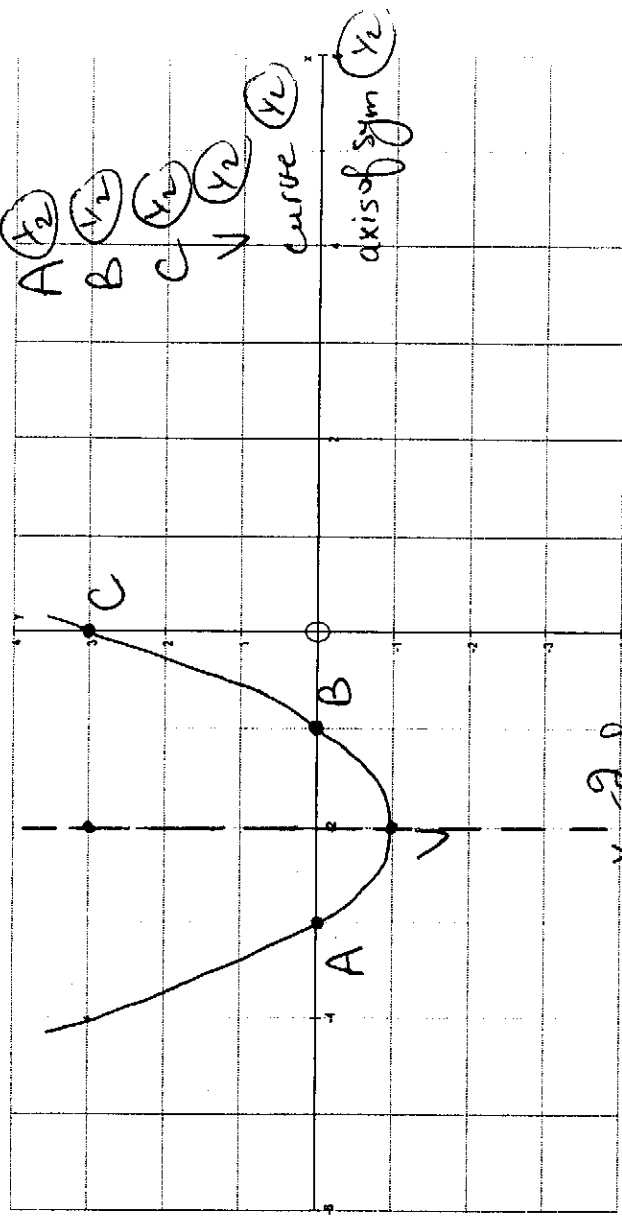
Graph the piece-wise function

$$f(x) = \begin{cases} -x - 1 & -2 \leq x < -1 \\ 2 & x = -1 \\ 1 - x^2 & -1 < x \leq 2 \\ -1 & x > 2 \end{cases}$$

4 pieces each ②



- VII. Consider the quadratic function  $y = f(x) = x^2 + 4x + 3$ .
- (2%) Determine whether the parabola opens up or down.
  - (4%) Find the coordinates of the vertex V.
  - (2%) Write the equation of the axis of symmetry.
  - (5%) Find x-intercept(s) and y-intercept.
  - (3%) Plot the graph.



$x = -2$  axis of sym

$a = 1$   
 $b = 4$   
 $c = 3$

a)  $a = 1 > 0$  ①  
 U-shape ①  
 opens up

b)  $x_v = -\frac{b}{2a} = -\frac{4}{2} = -2$  ①  
 $y_v = f(-2) = 4 + 4(-2) + 3 = -1$  ②  
 $V(-2, -1)$  ②

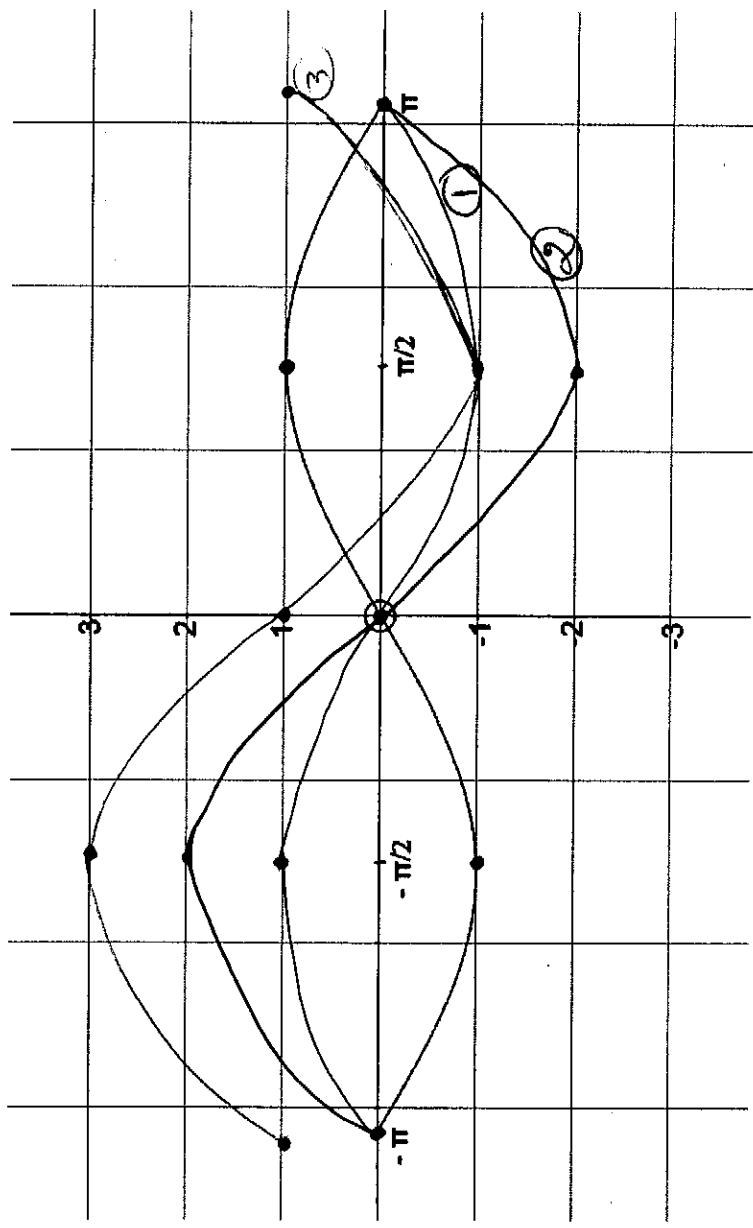
c)  $x = -\frac{b}{2a} = -2$  ①  
 $x = -2$  axis of symmetry

d) x-intercepts ①  
 $y = 0$   
 $x^2 + 4x + 3 = 0$   
 $(x+3)(x+1) = 0$   
 $x = -3$  [or]  $x = -1$   
 $A(-3, 0)$  ①  $B(-1, 0)$  ①

y-intercept  
 $x = 0$   
 $y = 3$   
 $C(0, 3)$  ①

VIII. Given the graph of  $f(x) = \sin x$  over the interval  $[-\pi, \pi]$ .

- Write down an expression for the graph of  $f(x)$  reflected with respect to x-axis then vertically stretched by a factor 2 then shifted 1 unit up.
- Sketch all steps.
- What is the range of the resulting function?



② each graph

- $y = -\sin x = -f(x)$  ②
- $y = 2(-\sin x) = -2 \sin x = -2 f(x)$  ②
- $y = -2 \sin x + 1 = -2 f(x) + 1$  ②

c)  $R = [-1, 3]$  ③  
 $-1 \leq y \leq 3$